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NONSTEADY PROPERTIES OF COUETTE FLOW OF A LIQUID UNDER THE CONDITIONS OF A PHASE TRANSITION

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Liquid flow under the conditions of the concurrent interaction of dissipative heat release and a phase transition was investigated in [1, 2]. In this case a quasi-steady approximation with respect to velocity and temperature was used, making it possible to determine the regions of the characteristic flow regimes: a complete phase transition, a regime of steady flow with the phase interface at an intermediate position, and a regime of hydrodynamic thermal explosion (HTE) [3].

Such an approach, presuming a sufficiently great heat of the phase transition and that the initial temperature and velocity distributions belong to the region of attraction of steady-state profiles, has a limited applicability. A clarification of the region of its applicability — the problem of nonsteady analysis — is discussed in the present paper.

1. Statement of the Problem

We consider the Couette flow of a viscous incompressible liquid lying between two coaxial infinite cylinders; the inner one (with a radius r_0) rotates while the outer one (with a radius r_1) is stationary. The outer cylinder is cooled below the temperature T_* of the phase transition, as a result of which a layer of solid material of thickness $\Delta = r_* - r_0$ is formed, where r_* is the coordinate of the phase interface. The Arrhenius temperature dependence of the viscosity $\eta = \eta_0 \exp(E/RT)$ is adopted, where E is the activation energy of the viscous flow, R is the universal gas constant, η_0 is a preexponential factor, and T is the temperature.

The system of equations of heat conduction and motion and the rheological equation can be written in the form

$$r < r_*:\ c_1\rho_1 \frac{\partial T}{\partial t} = \lambda_1 \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} \right) + \sigma r \frac{\partial \Omega}{\partial r}; \tag{1.1}$$

$$\frac{\partial \Omega}{\partial t} = \frac{1}{\rho_1 r^3} \frac{\partial}{\partial r}(\sigma r^2),\quad \sigma = \eta r \frac{\partial \Omega}{\partial r}; \tag{1.2}$$

$$r > r_*:\ c_2\rho_2 \frac{\partial T}{\partial t} = \lambda_2 \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} \right). \tag{1.3}$$

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Here Ω is the angular velocity of the liquid, t is time, r is the current radius, and c_1, c_2, ρ_1, ρ_2, λ_1, and λ_2 are the heat capacity, density, and coefficient of thermal conductivity of the liquid and solid phases, respectively.

It is assumed that at the initial time the liquid is at rest at a constant temperature T_i:

$$t=0:\Omega=0,\ T=T_i.$$

At the surface of the outer cylinder ($r = r_1$) we adopt the cooling law

$$\begin{aligned} &T>T_*:\ \lambda_1\frac{\partial T}{\partial r}=-\alpha_1(T-T_0),\\ &T<T_*:\ \lambda_2\frac{\partial T}{\partial r}=-\alpha_1(T-T_0),\\ &T_0=T_\infty+(T_i-T_\infty)\exp(-wt), \end{aligned} \tag{1.4}$$

where α_1 is the coefficient of thermal conductivity of the outer cylinder, T_∞ is the temperature of the surrounding medium, T_0 is the temperature of the outer cylinder, and w is the cooling rate. Such a cooling regime is easily accomplished in a nonisothermal rotary viscosimeter.

At the inner cylinder ($r = r_0$) we assign heat exchange by Newton's law,

$$\lambda_1=\partial T/\partial r=\alpha_2(T-T_s), \tag{1.5}$$

where α_2 and T_s are the heat-transfer coefficient and temperature of the inner cylinder.

A viscosimetric experiment is usually carried out either on a constant-velocity vicosimeter, when the rotational velocity of the inner cylinder is maintained, or on a constant-moment viscosimeter, when a constant deformation stress is maintained on the inner cylinder. Therefore, the equation of motion (1.2) is analyzed either for a constant velocity,

$$r=r_0:\Omega=\Omega_0, \tag{1.6}$$

or for an assigned stress,

$$r=r_0:\sigma=\sigma_0. \tag{1.7}$$

The conditions at the moving phase interface have the form

$$r=r_*:\ \Omega=0,\quad T=T_*,\quad \lambda_1\frac{\partial T}{\partial r}\Big|_{r=r_*-0}=\lambda_2\frac{\partial T}{\partial r}\Big|_{r=r_*+0}-Q\rho_1\frac{\partial r_*}{\partial t}, \tag{1.8}$$

where Q is the heat of the phase transition.

We introduce the dimensionless variables

$$\theta=\frac{E}{RT_*^2}(T-T_*);\quad \xi=\frac{r}{r_0},\quad \xi_*=\frac{r_*}{r_0},\quad \tau=\frac{ta_1}{r_0^2},$$

$\omega = \Omega/\Omega_0$ for an assigned velocity, and $\omega = \Omega\eta(T_*)/\sigma_0$ for an assigned stress.

In the dimensionless variables we write Eqs. (1.1)-(1.3) with the boundary conditions (1.4)-(1.8) in the form

$$\xi<\xi_*:\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial\theta}{\partial\xi}+\delta\left(\xi\frac{\partial\omega}{\partial\xi}\right)^2\exp\left(-\frac{\theta}{1+\beta\theta}\right); \tag{1.9}$$

$$\frac{\partial\omega}{\partial\tau}=\mathrm{Pr}\frac{1}{\xi^3}\frac{\partial}{\partial\xi}\left[\xi^3\exp\left(-\frac{\theta}{1+\beta\theta}\right)\frac{\partial\omega}{\partial\xi}\right]; \tag{1.10}$$

$$\xi>\xi_*:\frac{1}{a}\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial\theta}{\partial\xi}; \tag{1.11}$$

$$\xi=\xi_*:\theta=0,\ \omega=0,\ \frac{\partial\theta}{\partial\xi}\Big|_{\xi=\xi_*-0}=\lambda\frac{\partial\theta}{\partial\xi}\Big|_{\xi=\xi_*+0}-\Lambda\frac{\partial\xi_*}{\partial\tau}; \tag{1.12}$$

$$\xi=1:\frac{\partial\theta}{\partial\xi}=\mathrm{Bi}_2(\theta-\theta_s); \tag{1.13}$$

$$\omega=1\ \text{ or }\ \exp\left(-\frac{\theta}{1+\beta\theta}\right)\xi\frac{\partial\omega}{\partial\xi}=1; \tag{1.14}$$

$$\xi = d: \begin{cases} \theta > 0, & \frac{\partial\theta}{\partial\xi} = -\mathrm{Bi}_1 [\theta - \theta_\infty - (\theta_i - \theta_\infty)\exp(-\bar{w}\tau)], \\ \theta < 0, & \lambda\frac{\partial\theta}{\partial\xi} = -\mathrm{Bi}_1 [\theta - \theta_\infty - (\theta_i - \theta_\infty)\exp(-\bar{w}\tau)]. \end{cases} \tag{1.15}$$

Here $\lambda = \frac{\lambda_2}{\lambda_1}$; $a = \frac{a_2}{a_1}$; $d = \frac{r_1}{r_0}$; $\beta = \frac{RT_*}{E}$;

$$\theta_\infty = \frac{E}{RT_*^2}(T_\infty - T_*);\ \theta_s = \frac{E}{RT_*^2}(T_s - T_*);\ \theta_i = \frac{E}{RT_*^2}(T_i - T_*);$$

$$\bar{w} = \frac{r_0^2}{a_1}w;\ \mathrm{Bi}_1 = \frac{\alpha_1 r_0}{\lambda_1};\ \mathrm{Bi}_2 = \frac{\alpha_2 r_0}{\lambda_1};$$

$\Lambda = \frac{QF}{RT_*^2 c_1}$ and $\mathrm{Pr} = \frac{c_1\eta(T_*)}{\lambda_1}$ are dimensionless parameters. The parameter δ has the form

$$\delta = \frac{E}{RT_*^2}\,\frac{r_0^2\Omega_0^2\eta(T_*)}{\lambda_1}$$

for an assigned velocity and

$$\delta = \frac{E}{RT_*^2}\,\frac{r_0^2\sigma_0^2}{\lambda_1\eta(T_*)}$$

for an assigned stress.

2. Steady States of the System

Analytic solutions of the steady-state problem of liquid flow between two plane-parallel plates and pressurized flow in a pipe under the conditions of a phase transition were found in [1, 2]. The results of the solution of these problems are in qualitative agreement with the results of the solution of the steady-state problem of Couette flow between two coaxial cylinders, but the specific geometry of such flow introduces certain peculiarities. In particular, the parameter d characterizing the curvature appears here.

Let us consider states of the system when the inner cylinder is thermally insulated while heat exchange by Newton's law $w \to \infty$ is assigned at the outer cylinder. Performing a Frank–Kamenetskii transformation of the exponential ($\beta\theta \ll 1$, $\beta\theta^2 \ll 1$) [4], we obtain the steady-state system of equations in the form

$$\xi < \xi_*:\ \frac{d^2\theta}{d\xi^2} + \frac{1}{\xi}\frac{d\theta}{d\xi} + \delta\exp(-\theta)\left(\xi\frac{d\omega}{d\xi}\right)^2 = 0; \tag{2.1}$$

$$\frac{d}{d\xi}\left[\xi^3\frac{d\omega}{d\xi}\exp(-\theta)\right] = 0; \tag{2.2}$$

$$\xi > \xi_*:\ \frac{d^2\theta}{d\xi^2} + \frac{1}{\xi}\frac{d\theta}{d\xi} = 0; \tag{2.3}$$

$$\xi = \xi_*:\ \theta = 0,\ \omega = 0,\ \left.\frac{d\theta}{d\xi}\right|_{\xi=\xi_*-0} = \lambda\left.\frac{d\theta}{d\xi}\right|_{\xi=\xi_*+0}; \tag{2.4}$$

$$\xi = 1:\ \frac{d\theta}{d\xi} = 0,\ \omega = 1 \quad \text{or} \quad \xi\frac{d\omega}{d\xi}\exp(-\theta) = 1; \tag{2.5}$$

$$\xi = d:\ \frac{d\theta}{d\xi} = -\mathrm{Bi}_1(\theta - \theta_\infty). \tag{2.6}$$

After eliminating the velocity gradient with the help of (2.2), we reduce Eq. (2.1) to the well-known equation from the theory of a thermal explosion [4], after which we find the solution of the system (2.1)-(2.3) with the boundary conditions (2.4)-(2.6):

$$\xi < \xi_*:\ \theta = \ln(b_1\xi^2) - \ln\{\mathrm{ch}^2[b_2 - \sqrt{b_1 b_3\delta/2}\ln(\sqrt{\xi_*/\xi})]\}, \tag{2.7}$$

$$\omega = 1 - \sqrt{2b_1/\delta}\,\{\mathrm{th}(b_2 - \sqrt{b_1 b_3\delta/2}\ln\sqrt{\xi_*}) - \mathrm{th}[b_2 - \sqrt{b_1 b_3\delta/2}\ln(\sqrt{\xi_*/\xi})]\};$$

$$\xi > \xi_*:\ \theta = \frac{\mathrm{Bi}_1\theta_\infty\ln(\xi/\xi_*)}{1/d + \mathrm{Bi}_1\ln(d/\xi_*)}. \tag{2.8}$$

The constants of integration b_1 and b_2 and the coordinate ξ_* of the phase front are determined from the system of equations

$$\mathrm{ch}[b_2 + \sqrt{b_1 b_3\delta/2}\ln\sqrt{\xi_*}] = \sqrt{b_1\xi_*}; \tag{2.9}$$

$$\operatorname{th}\left[b_2+\sqrt{\frac{b_1b_3\delta}{2}}\ln\sqrt{\xi_*}\right]=\sqrt{\frac{2}{b_1b_3\delta}}\left\{1-\frac{\lambda\,\mathrm{Bi}_1\theta_\infty}{2\left[1/d+\mathrm{Bi}_1\ln(d/\xi_*)\right]}\right\};\tag{2.10}$$

$$\operatorname{th}\left[b_2-\sqrt{b_1b_3\delta/2}\ln\sqrt{\xi_*}\right]=\sqrt{2/(b_1b_3\delta)}.\tag{2.11}$$

The constant of integration b_3 has the meaning of the dimensionless stress at the inner cylinder. For an assigned stress $b_3 = 1$, while for an assigned velocity

$$b_3=\frac{1}{\delta}\,\frac{\lambda\,\mathrm{Bi}_1\theta_\infty}{1/d+\mathrm{Bi}_1\ln(d/\xi_*)}.$$

The system of equations (2.9)-(2.11) was solved numerically. We considered the limiting case of $\mathrm{Bi}_1 \to \infty$, i.e., a constant temperature T_∞ was assigned at the outer cylinder. The influence of the parameter Bi_1 on the steady states of the system during Couette flow was studied in [2].

It turned out (cf. [2]) that whereas for an assigned velocity, $\delta(\xi_*)$ is a monotonically increasing function, for an assigned stress this function proves to be nonmonotonic in a certain range of variation of the parameter $s = -\lambda\theta_\infty/2$ (Fig. 1, $s = 0.05$, $d = 1.1$). The boundaries $\delta_+(s)$ and $\delta_-(s)$ of the nonmonotonic region, determining the critical conditions for an HTE and for a complete phase transition, are shown in Fig. 2 (the solid lines correspond to $d = 1.1$). At $s = s_*$ the curves $\delta_+(s)$ and $\delta_-(s)$ merge, while at $s > s_*$ the function $\delta(\xi_*)$ becomes monotonically increasing. These results are in qualitative agreement with the results for the case of a plane-parallel strip. One can also show their quantitative agreement if the gap between the cylinders is small, i.e., $d - 1 \ll 1$; then the connection between the values of $\delta_\pm$ for coaxial cylinders and $\delta_\pm^{\parallel}$ for a plane-parallel strip [2] has the form

$$\delta_\pm=\delta_\pm^{\parallel}\,d^2/(1-d)^2.\tag{2.12}$$

A numerical investigation of Eqs. (2.9)-(2.11) showed that Eq. (2.12) yields an error of no more than 15% for $d < 1.5$. From (2.12) and the solution (2.9)-(2.11) it is seen that the critical values of δ decrease sharply with an increase in the layer between the cylinders. In Fig. 2 the dash-dot lines show $\delta_\pm(s)$ for $d = 1.5$.

The decrease in $\delta_\pm$ with an increase in d is explained by the fact that with an increase in the thickness of the liquid layer (and in the curvature), the conditions of heat transfer from the liquid are worsened, facilitating the development of an HTE.

We use the result of the analysis of [2] in the quasi-steady approximation, when the conditions

$$\Lambda\gg 1,\ \Pr\Lambda\gg 1\tag{2.13}$$

are satisfied. In [2] it is noted that only the increasing branch of the function $\delta(\xi_*)$ (see Fig. 1) can be stable. An analysis of the equation of motion of the phase front in the quasi-steady approximation with the initial conditions $\xi_* = 1$ for $\tau = 0$ enabled us to determine the region of characteristic states of the system in the plane of the parameters δ and s. The equation of the boundary $\delta_0(s)$ separating the region of flow with an intermediate position of the phase front from the region of an HTE with $s < s_0$ and the region of a complete phase transition from the region of an HTE with $s > s_0$ has the form (see Fig. 2).

$$\delta_0=0{,}878d^2(1+s)^2/(1-d)^2.\tag{2.14}$$

The curve $\delta_-(s)$ is the boundary separating the regions of a complete phase transition and of flow with an intermediate position of the phase front for $s < s_0$ (see Fig. 2).

3. Region of Applicability of the Quasi-Steady Approximation

The numerical solution of the problem (1.9)-(1.15) shows that the quasi-steady approximation $\Lambda = \Lambda_0$ is satisfied for intermediate values of the heat of the phase transition (Fig. 3), not too large but not too small. This can be explained as follows. For a low heat Q of the phase transition [when the condition (2.13) is not satisfied], quasi-steady temperature and velocity profiles will be unable to readjust under the rapid motion of the phase front. For a high heat Q in the system, even for subcritical δ ($\delta < \delta_0$), explosive superheating corresponding to the right-hand unstable branch of the function $\delta(\xi_*)$ can develop owing to the presence of an additional heat source due to the phase transition. The critical conditions for the development of nonsteady processes depend on the heat of the phase transition, and therefore Eq. (2.14) is satisfied not in some interval of variation of Λ, but at $\Lambda = \Lambda_0$. In Fig. 3 we present the dependence of Λ_0 on the parameter s characterizing the cooling intensity.

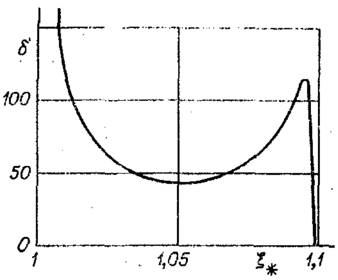

Fig. 1

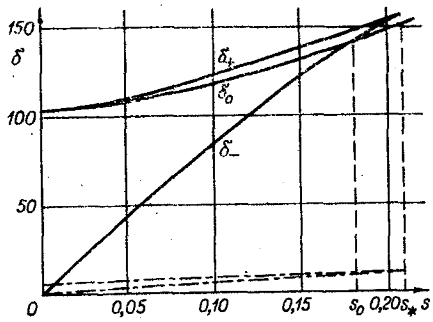

Fig. 2

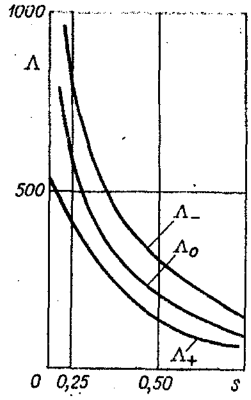

Fig. 3

In many cases it may prove necessary to calculate the critical conditions for an arbitrary Λ. The question of the applicability of Eq. (2.14) in a wide range of values of Λ is interesting in this connection. We calculated the region in which Eq. (2.14) is satisfied to within 10%. The boundaries of this region are the curves $\Lambda_+(s)$ and $\Lambda_-(s)$ (see Fig. 3) along which the critical values of δ ($\delta_0 \pm 0.1\delta_0$, respectively) are valid. It turned out that this region is rather broad, especially for small s [$\Lambda_+(s)/\Lambda_0(s) = 0.7$, $\Lambda_-(s)/\Lambda_0(s) = 2.4$ for $s = 0.2$]. With an increase in s, the region bounded by the curves $\Lambda_+(s)$ and $\Lambda_-(s)$ narrows somewhat, but still remains rather broad [$\Lambda_+(s)/\Lambda_0(s) = 0.7$ and $\Lambda_-(s)/\Lambda_0(s) = 1.2$ for $s = 0.75$]. Consequently, Eq. (2.14), obtained from the quasi-steady approximation, can be used with a small error in a wide range of variation of the parameter Λ (from Λ_+ to Λ_-; see Fig. 3).

4. Influence of the Initial Temperature on the Development of Nonsteady Processes

The investigation of the problem of a phase transition under the conditions of dissipative heat release in the steady-state and quasi-steady statements does not enable us to examine the influence of the initial temperature on the development of nonsteady processes. At the same time, it was pointed out in [2] that initial heating can greatly alter the critical conditions for the development of an HTE and a complete phase transition, obtained from the steady-state and quasi-steady theory. It is well known from the theory of a thermal explosion [5] that for an initial degree of heating greater than one, the critical value of δ at which a thermal explosion develops is greatly decreased. A critical value of δ, obtained from the steady-state theory of a thermal explosion [4], occurs only for a degree of heating $\theta < 1$. It can be expected that the influence of the initial degree of heating on the development of an HTE will be similar. It is also clear that the initial degree of heating can also influence the process of a complete phase transition. The hot liquid should solidify more slowly, i.e., for smaller δ, and therefore the region in Fig. 2 corresponding to a complete phase transition must become narrower.

In fact, the calculation shows that an increase in the initial temperature leads to a decrease in the critical value δ_0 obtained from the quasi-steady theory (see Fig. 4, where $s = 0.3$ and $\Lambda = 400$). This effect is less well manifested than in the theory of a thermal explosion, however. It is seen that δ_0 does not vary significantly in the interval of initial degrees of heating from 0 to 0.5. Even for $\theta_i = 1$ the critical value δ_0 decreases by only 20% [$\delta_0(1) = 140$]. The initial degree of heating has a significant influence only for $\theta_i > 1.5$.

Thus, the quasi-steady theory is applicable in a wide range of initial temperatures.

5. Nonsteady Peculiarities of the Development of an HTE and of a Complete Phase Transition

Let us consider the nonsteady peculiarities of the development of the processes during nonisothermal liquid flow under the conditions of a phase transition.

The calculation shows that two stages can be distinguished in the development of an HTE, differing in the direction of motion of the phase boundary (Fig. 5, curve 1). In Fig. 5

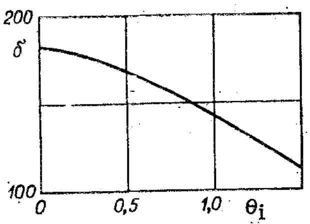

Fig. 4

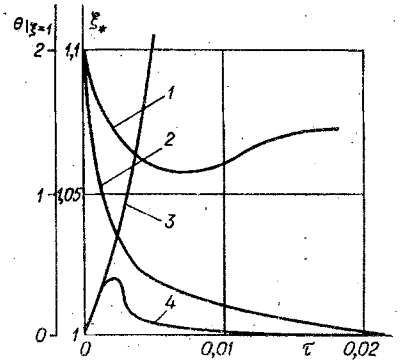

Fig. 5

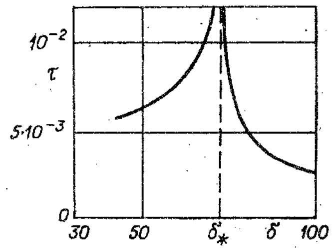

Fig. 6

curves 1 and 3 correspond to $s = 0.75$, $\delta = 319$, and $\Lambda = 115$ and curves 2 and 4 correspond to $s = 0.75$, $\delta = 319$, and $\Lambda = 10$. In the first stage ($\tau < \tau_*$) a layer of solid material forms on the cold wall (ξ_* decreases). The degree of heating in the liquid subsequently increases and in the second stage this layer starts to thaw (ξ_* increases, i.e., $\partial\xi_*/\partial\tau$ changes sign). The thickness of the layer formed in the first stage can reach about half the gap. In both stages of development of an HTE the temperature $\theta|_{\xi=1}$ of the adiabatic wall increases monotonically.

Two stages can also be distinguished in the process of a complete phase transition: In the first stage the temperature of the adiabatic wall rises while in the second stage it falls.

The change in the sign of $\partial\theta/\partial\tau|_{\xi=1}$ during a complete phase transition and in the sign of $\partial\xi_*/\partial\tau$ in an HTE is a characteristic feature of these processes, making it possible to distinguish them in early stages of development. Curves 1 and 2 show the time dependence of the position of the phase front during the development of an HTE and during a complete phase transition and curves 3 and 4 show the variation of the temperature of the inner cylinder during the development of an HTE and during a complete phase transition. The time of development of the nonsteady processes depends on the intensity of heat release and cooling, as well as on the initial conditions. In Fig. 6 we present the dependence of the period of induction of an HTE and of the time of a complete phase transition on the parameter δ, characterizing the intensity of heat release, for $\theta_i = 3$ and $s = 0.05$. For small δ a complete phase transition occurs in a short time. The time of a complete phase transition grows with an increase in δ and becomes infinite as $\delta \to \delta_*$. A hydrodynamic thermal explosion develops with a further increase in δ, with the period of induction of the HTE decreasing. The dependence of the times of the nonsteady processes on the initial temperature is similar.

The regime of an assigned rotational velocity, in which a unique steady state with the phase boundary at an intermediate position is always established (cf. [2]), and hence it is of little interest from the point of view of nonsteady peculiarities of flow, is mentioned only in passing in the present paper.

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